(8 pages)

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M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2020.

Fourth Semester

Mathematics — Core

TOPOLOGY — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. A space for which every open covering contains a countable sub covering is called

   (a) Separable (b) Compact

   (c) Lindelöf (d) Second countable

2. Which one of the following is not true?

(a) T_2 and compact $\Rightarrow$ normal

(b) T3 and Lindelöf $\Rightarrow$ $T_{3\frac{1}{2}}$

(c) T_2 and compact $\Rightarrow T_3$ and Lindelöf

(d) T_2 and compact $\Leftarrow T_3$ and Lindelöf

3. Every regular space with a countable basis is

(a) normal

(b) completely regular but not normal

(c) regular but not completely regular

(d) compact and Hausdorff

4. A space X is completely regular then it is homeomorphic to a subspace of

(a) $[0, 1]^J$

(b) $\mathbf{R}^n$ where n is a finite

(c) $\mathbf{R}^J$

(d) $(0, 1)^J$ where n is a finite number and J is uncountable.

5. Normal space is also known as

(a) T_4

(b) $T_{2\frac{1}{2}}$

(c) $T_{3\frac{1}{2}}$

(d) T_3

6. Tietze extension theorem implies

(a) The Urysohn Metrization theorem

(b) Heine-Borel theorem

(c) The Urysohn lemma

(d) The Tychonof theorem

7. Let $A = \{(n-1, n+1) : n \in Z\}$. Which of the following refine A.

(a) $\left\{\left(n-\frac{1}{2}, n+\frac{3}{2}\right) : n \in Z_+\right\}$

(b) $\left\{\left(n+\frac{1}{2}, n+\frac{3}{2}\right) : n \in Z_+\right\}$

(c) $\left\{\left(n-\frac{1}{2}, n+2\right) : n \in Z_+\right\}$

(d) $\{(x, x+1) : x \in R\}$

8. Which one of the following is locally finite in R?

(a) $\{(n-1, n+1) : n \in Z\}$

(b) $\left\{\left(0, \frac{1}{n}\right) : n \in Z_+\right\}$

(c) $\left\{\left(\frac{1}{n+1}, \frac{1}{n}\right) : n \in Z_+\right\}$

(d) $\{(x, x+1) : x \in R\}$

9. Which of the following is not true?

(a) Every non empty subset of the set of irrational numbers is of second category

(b) Open subspace of a Baire space is a Baire space

(c) The set of rationals is a Baire space

(d) If $X = \bigcup_{n=1}^{\infty} B_n$ and X is a Baire space with $B_1 \neq \phi$, then atleast one of $\overline{B_n}$ has nonempty interior.

10. Which one of the following is not true?

(a) Any set X with discrete topology is a Baire space

(b) Every locally compact space is a Baire space

(c) [0, 1] is a Baire space

(d) Rationals as a subspace of real numbers is not a Baire space

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a topological space. Let one point sets in X be closed. Then prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$.

Or

(b) Show that if X is regular, every pair of points of X have neighborhoods whose closures are disjoint.

12. (a) Examine the proof of Urysohn lemma and show that for a given r,

$$f^{-1}(r) = \left(\bigcap_{p>r} U_p - \bigcup_{q<r} U_q \right),$$ where p and q are rational.

Or

(b) Prove that every normal space is completely regular and completely regular space is regular.

13. (a) Prove that Tietze extension theorem implies the Urysohn lemma.

Or

(b) State and prove imbedding theorem.

14. (a) Give an example of a collection of sets A that is not locally finite, such that the collection $B = \{\overline{A} : A \in A\}$ is locally finite.

Or

(b) Define finite intersection property. Let X be a set and D be the set of all subsets of X that is maximal with respect to finite intersection property. Show that

(i) $x \in \overline{A} \forall A \in D$ if and only if every neighborhood of x belongs to D.

(ii) Let $A \in D$. Then prove that $B \supset A \Rightarrow B \in D$.

15. (a) Define a first category space. Prove that X is a Baire space if and only if 'given any countable collection $\{U_n\}$ of open sets in X, U_n is dense in X $\forall n$, then $\cap U_n$ is also dense'.

Or

(b) Define a Baire space. Whether Q the set of rationals as a space is Baire space? What about if we consider Q as a subspace of real numbers space. Justify your answer.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the space $\mathbf{R}_L$ satisfies all the countability axioms but the second.

Or

(b) Prove that product of Lindelof spaces need not be Lindelof.

17. (a) Define a regular space and a normal space. Prove that every regular second countable space is normal.

Or

(b) State and prove Urysohn's lemma.

18. (a) State and prove Tietze extension theorem.

Or

(b) State and prove Uryzohn's metrization theorem.

19. (a) State and prove Tychonoff theorem.

Or

(b) Let X be a metrizable space. If A is an open covering of X, then prove that there is an open covering ξ of X refining A that is countably locally finite.

20. (a) Let X be a space ; let (Y, d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$, where $f : X \to Y$. If X is a Baire space, prove that the set of points at which f is continuous is dense in X.

Or

(b) State and prove Baire Category Theorem.

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